

Online Appendix

for “Higher-Order Beliefs and Risky Asset Holdings” by Yuriy Gorodnichenko and Xiao Yin

I. Additional results

Table A.1: Summary Statistics Conditional on Both Waves

	Mean	SD	Mean	SD	Mean	SD	<i>p</i> -values	Mean	SD	<i>p</i> -values
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Panel A: All		Panel B: Control		Panel C: Treatment 1			Panel D: Treatment 2		
Age	38.77	11.48	38.68	10.95	39.65	11.90	0.11	37.99	11.54	0.25
Female	0.39	0.49	0.37	0.48	0.41	0.49	0.13	0.40	0.49	0.33
Wealth (K)	355.46	616.62	363.31	639.59	361.59	624.43	0.96	341.37	584.81	0.50
Income (K)	75.42	66.28	75.23	64.17	76.83	72.00	0.65	74.22	62.39	0.77
Return	3.84	18.68	4.27	16.78	4.08	19.89	0.85	3.17	19.28	0.26
Financial%	0.50	0.32	0.51	0.32	0.49	0.32	0.15	0.49	0.31	0.23
Stock %	0.26	0.28	0.26	0.27	0.27	0.30	0.48	0.26	0.29	0.71
ETF %	0.18	0.25	0.18	0.25	0.17	0.24	0.15	0.18	0.25	0.92
Derivative %	0.02	0.06	0.02	0.05	0.02	0.06	0.72	0.02	0.06	0.89
Bond %	0.36	0.32	0.36	0.32	0.37	0.33	0.61	0.36	0.33	0.85
Pension %	0.12	0.25	0.12	0.25	0.12	0.25	0.80	0.12	0.25	0.88
Risky_F%	0.46	0.32	0.46	0.32	0.45	0.32	0.75	0.46	0.32	0.76
Risky%	0.23	0.23	0.23	0.24	0.22	0.23	0.26	0.22	0.23	0.43
First order beliefs										
E[Return]	3.70	5.18	3.72	4.93	3.84	5.24	0.67	3.53	5.37	0.49
E[Δ S&P500]	3.41	5.25	3.43	4.98	3.54	5.53	0.69	3.25	5.25	0.51
SD[Return]	5.64	3.58	5.70	3.38	5.60	3.73	0.61	5.62	3.62	0.69
SD[Δ S&P500]	6.52	3.36	6.55	3.15	6.44	3.47	0.52	6.55	3.47	0.98
Higher order beliefs										
E[Δ S&P500]	3.77	5.32	3.71	5.09	4.00	5.60	0.29	3.59	5.25	0.69
SD[Δ S&P500]	6.48	3.54	6.48	3.34	6.39	3.58	0.62	6.58	3.7	0.59
N	2151		725		712			714		

Note: This table reproduces Table 1 in the main text conditional on those who completed both waves of surveys.

Table A.2: The Effects of Information Treatments on Beliefs with Controls

	E[Port]	E[Port]	FOB	FOB	HOB	HOB
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Expectations						
T1	-1.02*** (0.18)	-0.39* (0.22)	-1.09*** (0.15)	-0.27 (0.19)	-1.19*** (0.19)	-0.33 (0.24)
T2	-0.35* (0.18)	0.16 (0.22)	-0.15 (0.15)	0.38** (0.19)	-1.30*** (0.18)	0.03 (0.23)
Prior		0.56*** (0.03)		0.54*** (0.03)		0.59*** (0.03)
T1 x Prior		-0.20*** (0.04)		-0.24*** (0.04)		-0.24*** (0.04)
T2 x Prior		-0.16*** (0.04)		-0.14*** (0.04)		-0.35*** (0.04)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	3183	3170	3166	3171	3165	3174
R ²	0.13	0.29	0.10	0.30	0.03	0.20
Panel B: Uncertainty						
T1	-1.90*** (0.19)	-1.57*** (0.32)	-2.25*** (0.17)	-2.24*** (0.37)	-2.52*** (0.20)	-1.86*** (0.39)
T2	-1.93*** (0.19)	-1.37*** (0.32)	-2.26*** (0.17)	-1.84*** (0.36)	-2.70*** (0.20)	-1.62*** (0.38)
Prior		0.53*** (0.04)		0.39*** (0.04)		0.55*** (0.04)
T1 x Prior		-0.05 (0.05)		0.01 (0.05)		-0.10* (0.06)
T2 x Prior		-0.08 (0.05)		-0.05 (0.05)		-0.17*** (0.05)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
N	3249	3225	3279	3269	3289	3268
R ²	0.07	0.19	0.08	0.15	0.08	0.18

Note: This table reproduces Table 4 while adding pre-experiment controls

Table A.3: The Effects of Information Treatments on Beliefs after 3 Months

	E[Port]	E[Port]	FOB	FOB	HOB	HOB
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Expectations						
T1	0.12 (0.19)	-0.09 (0.22)	0.15 (0.18)	0.17 (0.20)	-0.08 (0.20)	-0.25 (0.23)
T2	0.06 (0.19)	-0.09 (0.21)	0.21 (0.18)	0.31 (0.20)	-0.09 (0.20)	-0.10 (0.23)
Prior		0.31*** (0.03)		0.36*** (0.03)		0.26*** (0.03)
T1 x Prior		0.05 (0.04)		-0.05 (0.04)		0.02 (0.04)
T2 x Prior		0.06 (0.04)		-0.04 (0.04)		0.01 (0.04)
Controls	No	No	No	No	No	No
N	2137	2136	2151	2138	2151	2151
R ²	0.00	0.19	0.00	0.17	0.00	0.11
Panel B: Uncertainty						
T1	-0.36** (0.16)	-0.08 (0.23)	-0.02 (0.15)	-0.05 (0.27)	-0.12 (0.15)	0.25 (0.28)
T2	-0.29* (0.16)	0.20 (0.24)	-0.16 (0.15)	0.06 (0.28)	-0.33** (0.15)	0.43 (0.27)
Prior		0.61*** (0.03)		0.56*** (0.03)		0.57*** (0.03)
T1 x Prior		-0.05 (0.04)		0.02 (0.04)		-0.06 (0.04)
T2 x Prior		-0.09** (0.04)		-0.02 (0.04)		-0.12*** (0.04)
Controls	No	No	No	No	No	No
N	2151	2148	2151	2148	2151	2151
R ²	0.00	0.37	0.00	0.37	0.00	0.32

Note: This table reproduces Table 4 with left-hand side variables replaced with those from the second wave of surveys.

Table A.4: The Effects Beliefs on Asset Holding within Financial Assets

	Stock	ETF	Der	Bonds	Pension
	(1)	(2)	(3)	(4)	(5)
FOB	0.60	0.74	0.20	-2.48**	-0.49
	(0.58)	(0.58)	(0.14)	(1.23)	(0.72)
HOB	-0.27	-1.17***	-0.03	2.10**	0.63
	(0.40)	(0.45)	(0.09)	(0.95)	(0.61)
Controls	Yes	Yes	Yes	Yes	Yes
N	1990	1990	1989	1990	1990
First-stage <i>F</i> -stats					
FOB	18.54	18.54	18.54	18.54	18.54
HOB	17.41	17.41	17.41	17.41	17.41

Note: Stock%, ETF%, Der%, Bonds%, and Pension% are respectively the share of financial wealth invested in single companies, ETF and other index funds, financial derivatives, bonds, and pension. Controls are all pre-experiment and include prior expectations, risky asset share, sex, age, indicator for full-time employees, indicator for having at least college degree, ethnic group fixed effects, implied prior return volatilities, reaction speeds, log income, and portfolio returns. Outliers and influential observations are identified and removed according to the procedure described in Coibion et al. (2023). FOB and HOB are winsorized at 1% and 99% level. * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

Table A.5: Average Effects of Treatments on Asset Holdings

	Risk%	Risk%	Risk F%	Risk F%
	(1)	(2)	(3)	(4)
T1	-1.33	0.29	-1.26	0.35
	(0.85)	(0.56)	(0.79)	(0.54)
T2	0.63	1.01*	0.58	1.04*
	(0.86)	(0.56)	(0.79)	(0.54)
Controls	No	Yes	No	Yes
N	2151	2081	2151	2151

Note: Risky_F% is the share of financial assets invested in single stocks, ETF and index funds, and financial derivatives. Risky% is the product of Risky_F% and the share of financial assets. Controls are all pre-experiment and include prior expectations, pre-experiment risky asset allocations, sex, age, indicator for full-time employees, indicator for having at least college degree, ethnic group fixed effects, implied prior return volatilities, reaction speeds, log income, and portfolio returns. Regressions are based on Huber robust regressions. * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

Robert Shiller Investor Confidence Surveys Questions

C. (4) How much of a change in percentage terms do you expect [for the Dow Jones index] in the following 1 month? 3 months? 6 months? 1 year? 10 years?

D. (5) Although I expect a substantial drop in stock prices in the U.S. ultimately, I advise being relatively heavily invested in stocks for the time being because I think that prices are likely to rise for a while

1. True. Your best guess for date of peak
2. False.
3. No opinion.

E. (6) Although I expect a substantial rise in stock prices in the U.S. ultimately, I advise being less invested in stocks for the time being because I think that prices are likely to drop for a while

1. True. Your best guess for date of bottom
2. False.
3. No opinion.

F. (11) Many people are showing a great deal of excitement and optimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them

1. True.
2. False.
3. No opinion

G. (12) Many people are showing a great deal of pessimism about the prospects for the stock market in the United States, and I must be careful not to be influenced by them

1. True.
2. False.
3. No opinion

K. (16) Are you inclined now to buy stocks overall, or sell stock overall, or hold steady and why?

1. Buy.
2. Sell.
3. Hold

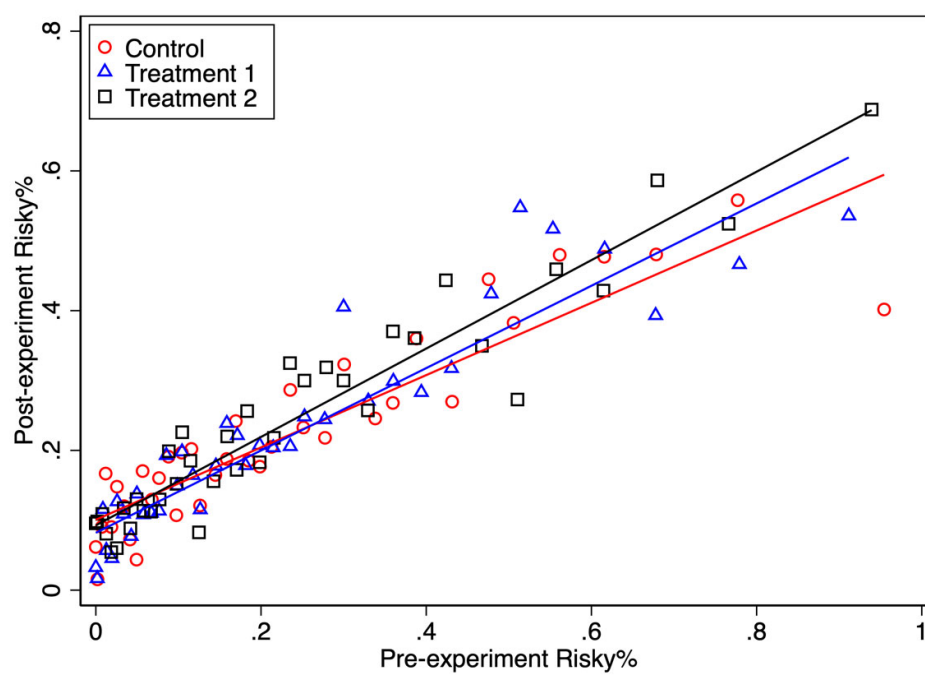
Why:

Table A.6: Higher-Order Beliefs and Buy-Hold Decisions – ICF Data

	Buy-Sell			
	(1)	(2)	(3)	(4)
FOB-1M	1.475*** (0.210)			
FOB-3M		2.105*** (0.139)		
FOB-6M			2.034*** (0.104)	
FOB-12M				1.872*** (0.072)
HOB	-0.076*** (0.006)	-0.071*** (0.006)	-0.065*** (0.006)	-0.058*** (0.005)
Date FE	Yes	Yes	Yes	Yes
R ²	0.04	0.06	0.09	0.12
N	6242	6343	6504	6834
	Buy-Sell-Speculative			
	(5)	(6)	(7)	(8)
FOB-1M	9.385*** (0.557)			
FOB-3M		8.688*** (0.343)		
FOB-6M			5.984*** (0.260)	
FOB-12M				3.729*** (0.184)
HOB	-0.071*** (0.015)	-0.046*** (0.015)	-0.042*** (0.015)	-0.043*** (0.015)
Date FE	Yes	Yes	Yes	Yes
R ²	0.05	0.10	0.08	0.07
N	6293	6392	6549	6877

Note: This table shows results of buy-sell preferences on higher-order beliefs using Robert Shiller Investor Confidence surveys. The questions are shown above and data is from Schmidt-Engelbertz and Vasudevan (2023). Buy-Sell is the answer to K. (16). Buy-Sell-Speculative is the difference between the answers to questions D. (5) and E. (6). HOB is the difference between the answers to questions F. (11) and G. (12). Following Engelbertz and Vasudevan (2023), we encode all answers of True/Buy with 1, False/Sell with -1, and No opinion/Hold with 0. FOB-1M to FOB-12M are respectively the answers to questions C. (4). The FOB variables, which are continuous, are winsorized at 1%-99% levels. All columns include survey-filling date fixed effects. Results are based on Huber robust regressions. * $p < 0.10$ ** $p < 0.05$ *** $p < 0.01$.

Figure A.1: Post-experiment and Pre-experiment Risky Asset Holdings



Note: Risky% is the product of share of financial assets invested in single stocks, ETF and index funds, and financial derivatives and the share of financial assets.

II. Proofs

a. Proof of Lemma 1 and Lemma 2

We will guess and verify that

$$\bar{E}[\tilde{r}_2 | s_i, s_{im}] = \kappa_D \tilde{r}_2. \quad (1)$$

Intuitively, the average belief is a linear function of the fundamental. We assume that

$$s_{im} = \kappa_D (\tilde{r}_2 + \eta_i),$$

with $\eta_i \sim N(0, \sigma_\eta^2)$ an idiosyncratic shock. Label $\sigma_{sim}^2 \equiv \text{var}(s_{im}) = \kappa_D^2 (\sigma_0^2 + \sigma_\eta^2)$. To simplify notation, we normalize $\sigma_0^2 = 1$ and let $\sigma_v^2 = \frac{\tau}{1-\tau} \sigma_0^2 = \frac{\tau}{1-\tau}$. Note that $0 < \tau < 1$ so that $\sigma_v^2 > 0$. These assumptions imply $\sigma_{s_i}^2 = \frac{1}{1-\tau}$. In addition, $\sigma_{sim}^2 \equiv \xi \kappa_D^2$ with $\xi = 1 + \sigma_\eta^2$.

The covariance structure of the random variables is

$$\begin{bmatrix} \tilde{r}_2 \\ s_i \\ s_{im} \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & \kappa_D \\ 1 & \frac{1}{1-\tau} & \kappa_D \\ \kappa_D & \kappa_D & \xi \kappa_D^2 \end{bmatrix} \right),$$

Conditional expectation of D of rational trader i given the two signals is

$$E_i[\tilde{r}_2] \equiv E[\tilde{r}_2 | s_i, s_{im}] = \begin{bmatrix} 1 & \kappa_D \end{bmatrix} \begin{bmatrix} \frac{1}{1-\tau} & \kappa_D \\ \kappa_D & \sigma_{sim}^2 \end{bmatrix}^{-1} \begin{bmatrix} s_i \\ s_{im} \end{bmatrix} = \kappa_s s_i + \kappa_{sm} s_{im},$$

where $\kappa_s = \frac{(1-\tau)(\xi-1)}{\xi-(1-\tau)}$, $\kappa_{sm} = \frac{\tilde{\kappa}}{\kappa_D}$, $\tilde{\kappa} \equiv \frac{\tau}{\xi-(1-\tau)}$.

This proves Lemma 2.

In addition,

$$\bar{E}[\tilde{r}_2 | s_i, s_{im}] = \kappa_s \tilde{r}_2 + \kappa_{sm} \kappa_D \tilde{r}_2 = (\kappa_s + \tilde{\kappa}) \tilde{r}_2 \quad (2)$$

Matching coefficients between (1) and (2) gives $\kappa_D = \kappa_s + \tilde{\kappa}$. Given $\xi > 1$ and $\tau \in (0,1)$, we have $\kappa_D \in (0,1)$. This proves Lemma 1.

We can also calculate the subjective variance of D given s_i and s_{im} , which is

$$\text{var}(\tilde{r}_2 | s_i, s_{im}) = 1 - \begin{bmatrix} 1 & \kappa_D \end{bmatrix} \begin{bmatrix} \frac{1}{1-\tau} & \kappa_D \\ \kappa_D & \xi \kappa_D^2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ \kappa_D \end{bmatrix} = \frac{\tau(\xi-1)}{\xi-(1-\tau)} = 1 - \kappa_D.$$

b. Proof of Lemma 3

The total demand in the market is

$$x = \frac{\bar{E}[\tilde{r}_2 | s_i, s_{im}, \tilde{r}_1] - \tilde{r}_1}{\gamma \hat{\sigma}^2}, \quad (3)$$

where x is the average portfolio share in t_1 , $\bar{x}$ is the average equity share in t_0 . $\hat{\sigma}^2 \equiv \text{var}(\tilde{r}_2|s_i, s_{im}, \tilde{r}_1)$.

(3) yields

$$\tilde{r}_1 = \bar{E}[\tilde{r}_2|s_i, s_{im}, \tilde{r}_1] - x \gamma \hat{\sigma}^2 \quad (4)$$

Define $\tilde{r}_2^0 \equiv \tilde{r}_2|s_i, s_{im}$. Based on Bayesian learning,

$$E[\tilde{r}_2^0 | \tilde{r}_1] = E[\tilde{r}_2^0] + \beta_P(\tilde{r}_1 - E_i[\tilde{r}_2|s_i, s_{im}]). \quad (5)$$

$$\begin{aligned} \beta_P &= \frac{\text{cov}(\tilde{r}_2^0, \tilde{r}_1)}{\text{var}(\tilde{r}_1)}, \\ \hat{\sigma}^2 &= (1 - \text{corr}(\tilde{r}_2^0, \tilde{r}_1)^2) \text{var}(\tilde{r}_2^0) \\ &= \text{var}(\tilde{r}_2^0) - \text{corr}(\tilde{r}_2^0, \tilde{r}_1)^2 \text{var}(\tilde{r}_2^0) \\ &= \text{var}(\tilde{r}_2^0) - \frac{\text{cov}(\tilde{r}_2^0, \tilde{r}_1)^2}{\text{var}(\tilde{r}_1)} \\ &= \text{var}(\tilde{r}_2^0) - \text{cov}(\tilde{r}_2^0, \tilde{r}_1) \beta_P. \end{aligned}$$

From (5),

$$\bar{E}[\tilde{r}_2|s_i, s_{im}, \tilde{r}_1] = \beta_P \tilde{r}_1 + (1 - \beta_P \kappa_D) \bar{E}[\tilde{r}_2|s_i, s_{im}]. \quad (6)$$

Combining (4) with (6) gives

$$\tilde{r}_1 = \frac{1 - \beta_P \kappa_D}{1 - \beta_P} \bar{E}[\tilde{r}_2|s_i, s_{im}] - \frac{x \gamma \hat{\sigma}^2}{1 - \beta_P}. \quad (7)$$

Conjecture

$$\tilde{r}_1 = A_1 + B_1 \tilde{r}_2 + C_1 x,$$

Note that $\tilde{r}_2^0 \sim N(\kappa_S s_i + \kappa_{sm} s_{im}, 1 - \kappa_D)$. Based on (1) and (7), we have

$$\tilde{r}_1 = \frac{1 - \beta_P \kappa_D}{1 - \beta_P} \kappa_D \tilde{r}_2 - \frac{x \gamma \hat{\sigma}^2}{1 - \beta_P}.$$

Therefore, $A_1 = 0$, $B_1 = \frac{1 - \beta_P \kappa_D}{1 - \beta_P} \kappa_D$, $C_1 = -\frac{x \gamma \hat{\sigma}^2}{1 - \beta_P}$.

Now we want to prove that $\beta_P < 1$. Since $\hat{\sigma}^2 > 0$, we have

$$\beta_P < \frac{\text{var}(\tilde{r}_2^0)}{\text{cov}(\tilde{r}_2^0, \tilde{r}_1)} = \frac{1 - \kappa_D}{\frac{1 - \beta_P \kappa_D}{1 - \beta_P} \kappa_D^2}. \quad (8)$$

First, suppose $1/\kappa_D > \beta_P \geq 1$, then from (6), $\beta_P \leq 0$, which is a contradiction. If $\beta_P = 1/\kappa_D$, the RHS of (6) becomes $-\infty$. Then suppose $\beta_P > 1/\kappa_D$, then the RHS of (6) is smaller than $1 - \kappa_D$. This again yields a contradiction. In the end, suppose $\beta_P < 0$, then $\text{cov}(\tilde{r}_2^0, \tilde{r}_1) > 0$. From (5), $\beta_P > 0$. This is a contradiction. Therefore, $\beta_P < 1$.

c. Proof of Proposition 1

The subjective payoff can be written as

$$\begin{aligned}
E_i[\tilde{r}_2|s_i, s_{im}, \tilde{r}_1] &= E_i[\tilde{r}_2|s_i, s_{im}] + \beta_p(\tilde{r}_1 - E_i[\tilde{r}_1|s_i, s_{im}]) \\
&= E_i[\tilde{r}_2|s_i, s_{im}] + \beta_p\tilde{r}_1 - \beta_p E_i[\bar{E}[\tilde{r}_2]|s_i, s_{im}].
\end{aligned}$$

The optimal holding can be written as

$$\begin{aligned}
x_i &= \frac{E[\tilde{r}_2|s_i, s_{im}, \tilde{r}_1] - \tilde{r}_1}{\gamma \hat{\sigma}^2} \\
&= \frac{E_i[\tilde{r}_2|s_i, s_{im}] - \beta_p E_i[\bar{E}[\tilde{r}_2]|s_i, s_{im}] - (1 - \beta_p)\tilde{r}_1}{\gamma \sigma^2}
\end{aligned}$$

d. Proof of Proposition 2

To characterize the heterogeneity in reaction speed, we further split t_1 into two sub-periods, $t_{1,1}$ and $t_{1,2}$. At the beginning of $t_{1,1}$. The timeline of t_1 can be summarized as follows:

1. At the beginning of $t_{1,1}$, A receives two signals: one about p_2 , the other about everyone else's average belief about p_2 . Based on these two signals, A forms a posterior view about p_2 .
2. In $t_{1,1}$, A speculates on the log equilibrium price $p_{1,2}$ at $t_{1,2}$ given subjective beliefs, and chooses the level of holding by maximizing utility over total wealth in $t_{1,2}$. In contrast, B holds zero dollars of the asset. The equilibrium price $P_{1,1}$ is realized.
3. In $t_{1,2}$, a random $1 - \tilde{\alpha}$ of A sell their asset. In addition, B receives two signals: one about p_2 , the other about everyone else's average belief about p_2 . Based on these two signals, B forms a posterior view about p_2 .
4. B chooses the holding level to maximize utility over subjective beliefs about p_2 . In addition, B uses the posteriors from Stage 3 as priors and learns about others' valuations from $p_{1,2}$, which is the log of the noisy rational expectation equilibrium (REE) price at the end of t_1 .

We first solve for the optimal equity share the given beliefs and then derive each investor's subjective beliefs. Let R_{t+1} be investor i 's final return of risky asset. For A in $t_{1,1}$, $R_{t+1} = P_{1,2}/P_{1,1} - 1$. For B in $t_{1,2}$, $R_{t+1} = P_2/P_{1,2} - 1$. The optimization problem for investor i yields

$$x_i = \frac{E_i[r_{t+1}]}{\gamma \sigma_i^2} = \frac{E_i[\tilde{r}_{t+1}] - \tilde{r}_t}{\gamma \sigma_i^2}, \quad (9)$$

where σ_i^2 is the conditional variance of r_{t+1} , $\tilde{r}_h = p_h - p_0$ is the log return from t_0 to t_h .

Integrating both sides of (9) gives

$$\tilde{r}_t = \int \frac{\sigma_i^{-2}}{\int \sigma_i^{-2} di} E_i[\tilde{r}_{t+1}] di - \frac{\gamma x}{\int \sigma_i^{-2} di}. \quad (10)$$

Hence, the equilibrium prices (scaled by p_0) is the subjective certainty-weighted average belief of all individuals minus the risk premium.

Equilibrium Prices: we first derive the equilibrium prices. In $t_{1,1}$, B is dormant and hold zero dollar of the asset. Then total demand in the market is

$$x = \alpha \frac{\bar{E}[\tilde{r}_{1,2}|s_i, s_{im}] - \tilde{r}_{1,1}}{\gamma V_A}, \quad (4)$$

where x is the average portfolio share in t_1 , $\bar{x}$ is the average equity share in t_0 . $V_A \equiv \text{var}(p_{1,2}|s_i, s_{im})$.

Denote $\alpha_1 \equiv \tilde{\alpha}\alpha$ as the total mass of investors who fail to sell their asset in $t_{1,2}$. In $t_{1,2}$,

$$x = \alpha_1 \frac{\bar{E}[\tilde{r}_{1,2}|s_i, s_{im}] - \tilde{r}_{1,1}}{\gamma V_A} + (1 - \alpha) \frac{\bar{E}[\tilde{r}_2|s_i, s_{im}, \tilde{r}_{1,2}] - \tilde{r}_{1,2}}{\gamma V_B}. \quad (5)$$

(4) and (5) yields

$$\tilde{r}_{1,1} = \bar{E}[\tilde{r}_{1,2}|s_i, s_{im}] - \frac{x}{\alpha} \gamma V_A \quad (6)$$

$$\tilde{r}_{1,2} = \bar{E}[\tilde{r}_2|s_i, s_{im}, \tilde{r}_{1,2}] - \frac{1 - \tilde{\alpha}}{1 - \alpha} x \gamma V_B \quad (7)$$

Define $\tilde{r}_2^0 \equiv \tilde{r}_2|s_i, s_{im}$. Based on Bayesian learning,

$$E[\tilde{r}_2^0 | \tilde{r}_{1,2}] = E[\tilde{r}_2^0] + \beta_{\tilde{P}}(\tilde{r}_{1,2} - E_i[p_{1,2}|s_i, s_{im}]). \quad (8)$$

$$\beta_{\tilde{P}} = \frac{\text{cov}(\tilde{r}_2^0, \tilde{r}_{1,2})}{\text{var}(\tilde{r}_{1,2})},$$

$$V_B = (1 - \text{corr}(\tilde{r}_2^0, \tilde{r}_{1,2})^2) \text{var}(\tilde{r}_2^0) = \text{var}(\tilde{r}_2^0) - \text{cov}(\tilde{r}_2^0, \tilde{r}_{1,2})\beta_{\tilde{P}}.$$

Based on the same logic, we have $0 < \beta_{\tilde{P}} < 1$.

From (8),

$$\bar{E}[\tilde{r}_2|s_i, s_{im}, \tilde{r}_{1,2}] = \beta_{\tilde{P}}\tilde{r}_{1,2} + (1 - \beta_{\tilde{P}}\kappa_D)\bar{E}[\tilde{r}_2|s_i, s_{im}]. \quad (9)$$

Combining (7) with (9) gives

$$\tilde{r}_{1,2} = \frac{1 - \beta_{\tilde{P}}\kappa_D}{1 - \beta_{\tilde{P}}} \bar{E}[\tilde{r}_2|s_i, s_{im}] - \frac{(1 - \tilde{\alpha}) \gamma V_B}{(1 - \alpha)(1 - \beta_{\tilde{P}})} x. \quad (10)$$

Based on (10),

$$E[\tilde{r}_{1,2}|s_i, s_{im}] = E\left[\frac{1 - \beta_{\tilde{P}}\kappa_D}{1 - \beta_{\tilde{P}}} \bar{E}[\tilde{r}_2|s_i, s_{im}] - \frac{(1 - \tilde{\alpha}) \gamma V_B}{(1 - \alpha)(1 - \beta_{\tilde{P}})} x | s_i, s_{im}\right]$$

Note that from the perspective of the investors before t_1 , $E[x|s_i, s_{im}] = 0$. Therefore

$$E[\tilde{r}_{1,2}|s_i, s_{im}] = \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} E[\bar{E}[\tilde{r}_2]|s_i, s_{im}] \quad (11)$$

Combining (6) and (11) gives

$$\tilde{r}_{1,1} = \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} \kappa_D \bar{E}[\tilde{r}_2|s_i, s_{im}] - \frac{\gamma V_A}{\alpha} x. \quad (12)$$

Conjecture

$$\tilde{r}_{1,1} = A_1 + B_1 \tilde{r}_2 + C_1 x,$$

$$\tilde{r}_{1,2} = A_2 + B_2 \tilde{r}_2 + C_2 x.$$

Note that $\tilde{r}_2^0 \sim N(\kappa_s s_i + \kappa_{sm} s_{im}, \tilde{\kappa} + \kappa_s)$. Based on (1) and (12), we have

$$\tilde{r}_{1,1} = \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} \kappa_D^2 \tilde{r}_2 - \frac{\gamma V_A}{\alpha} x$$

Therefore, $A_1 = 0$, $B_1 = \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} \kappa_D^2$, $C_1 = -\frac{\gamma V_A}{\alpha}$, $V_A = B_2^2 + C_2^2 \sigma_x^2$.

For $\tilde{r}_{1,2}$, based on (1) and (10), we have

$$\tilde{r}_{1,2} = \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} \kappa_D \tilde{r}_2 - \frac{(1 - \tilde{\alpha}) \gamma V_B}{(1 - \alpha)(1 - \beta_{\bar{p}})} x.$$

Therefore, $A_2 = 0$, $B_2 = \frac{B_1}{\kappa_D}$, $C_2 = -\frac{(1 - \tilde{\alpha}) \gamma V_B}{(1 - \alpha)(1 - \beta_{\bar{p}})}$.

Optimal Holding For those in A who can sell their asset in $t_{1,2}$, their net holding in t_1 is zero.

For the other investors, the average investor's holding is

$$\begin{aligned} x_i &= \frac{\alpha_1}{\gamma V_A} (E[\tilde{r}_{1,2}|s_i, s_{im}] - \tilde{r}_{1,1}) + \frac{1 - \alpha}{\gamma V_B} (E[\tilde{r}_2|s_i, s_{im}, \tilde{r}_{1,2}] - \tilde{r}_{1,2}). \\ &= \frac{\alpha_1}{\gamma V_A} \left(E \left[\frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} \bar{E}[\tilde{r}_2|s_i, s_{im}] | s_i, s_{im} \right] - \tilde{r}_{1,1} \right) \\ &\quad + \frac{1 - \alpha}{\gamma V_B} \left(E[\tilde{r}_2|s_i, s_{im}] - \beta_{\bar{p}} \frac{1 - \beta_{\bar{p}}\kappa_D}{1 - \beta_{\bar{p}}} E[\bar{E}[\tilde{r}_2]|s_i, s_{im}] - (1 - \beta_{\bar{p}}) \tilde{r}_{1,2} \right) \end{aligned} \quad (13)$$

After simplification, we get

$$x_i = \omega_0 + \omega_F E[\tilde{r}_2|s_i, s_{im}] + \omega_H E[\bar{E}[\tilde{r}_2]|s_i, s_{im}] \equiv \omega_0 + \omega_F E_i[\tilde{r}_2] + \omega_H E_i[\bar{E}[\tilde{r}_2]],$$

$$\omega_0 = -\frac{\alpha_1}{\gamma V_A} \tilde{r}_{1,1} - \frac{1 - \alpha}{\gamma V_B} (1 - \beta_{\bar{p}}) \tilde{r}_{1,2}$$

$$\omega_F = \frac{1 - \alpha}{\gamma V_B}$$

$$\omega_H = \frac{1 - \beta_{\bar{p}}\kappa_D}{\gamma (1 - \beta_{\bar{p}})} \left(\frac{\alpha_1}{V_A} - \frac{1 - \alpha}{V_B} \beta_{\bar{p}} \right).$$

An increase in FOB leads to more stock holding. The effect of higher HOB on stock holding is ambiguous. When $\alpha_1/V_A < (1 - \alpha) \beta_{\bar{P}}/V_B$, stock holding decreases with HOB. The sign depends on the composition of the investors, when everyone is B , $\alpha \rightarrow 0$, then $\omega_H \rightarrow -\frac{1}{\gamma V_B} \frac{1-\beta_P \kappa_D}{1-\beta_{\bar{P}}} \beta_{\bar{P}} < 0$. In contrast, when $\alpha_1 \rightarrow 1$, $\omega_H \rightarrow \frac{1}{\gamma V_A} \frac{1-\beta_{\bar{P}} \kappa_D}{1-\beta_{\bar{P}}} > 0$. This proves Proposition 1.

e. Proof of Proposition 3

In the model, F_i and H_i are positive correlated with covariance κ_D . We can write $H_i = \kappa_D F_i$. Then a univariate regression of x_i on F_i yields $x_i = \alpha_0 + \omega_0 F_i$, where $\omega_0 = \omega_F + \omega_H \kappa_D$. Suppose $\omega_H < 0$, then $\omega_0 < \omega_F$. The intuition is the same when studying the effects of HOB on stock holding while unconditional on FOB.

III. Questionnaires

a. First Wave Survey

What is the total level (\$) of your current wealth?

How long have you been investing in the stock market?

Never invested in the stock market.

Less than 1 year.

1 to 3 years.

3 to 5 years.

5-10 years

More than 10 years

Approximately what percentage of your current wealth is financial wealth?

Note: financial wealth includes stocks, ETFs, financial derivatives, bonds, pension funds, bank savings, and other wealth in the financial system.

0 10 20 30 40 50 60 70 80 90 100
%



500,000 - 1,000,000

Around how many times do you check the current value of your stock-market wealth every year?

0 20 40 60 80 100 120 140 160 180 200
Number of times



Around how many times do you change your wealth allocation to the stock market every year?

0 20 40 60 80 100 120 140 160 180 200

Number of times



We would now like to ask how your current financial assets (excluding real estate) are distributed across different asset classes. Please enter the approximate percentage you currently have invested in the following asset classes.

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

Stocks (Individual Companies)	0 %
ETFs or index fund	0 %
Financial derivatives (option, future, forward, etc)	0 %
Bonds	0 %
Pension fund (401k, IRA etc)	0 %
Other	0 %
Total	0 %

Over the **past** twelve months, by how much (in % changes relative to the current level) have your stock market portfolio changed?

Note: please use negative values for a decrease and positive values for an increase.

-100 -80 -60 -40 -20 0 20 40 60 80 100

%



Your total pre-tax earnings (\$) over the **past** 12 months were:

Note: earnings include wages, salaries, bonuses, commission, etc., excluding capital gains.

less than 5,000

5,000 - 10,000

10,000 - 25,000

25,000 - 50,000

50,000 - 75,000

75,000 - 100,000

100,000-150,000

150,000 - 250,000

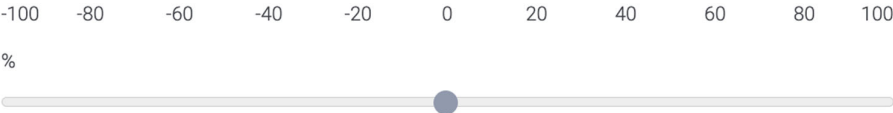
250,000 - 500,000

more than 500,000

Each of the following 4 questions are sent to a random 25% of participants

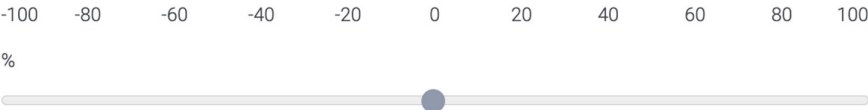
Suppose that the S&P500 index has increased by 5% over the past three months. How would you change your stock holdings.

e.g. if you would allocate 10% more of your wealth to the stock market, select 10%. If you would sell 10% of your stock market wealth, select -10%.



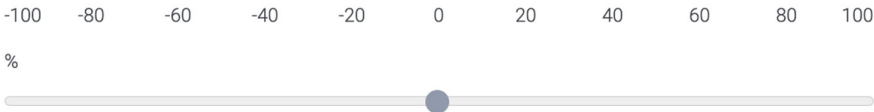
Suppose that the S&P500 index has increased by 10% over the past three months. How would you change your stock holdings.

e.g. if you would allocate 10% more of your wealth to the stock market, select 10%. If you would sell 10% of your stock market wealth, select -10%.



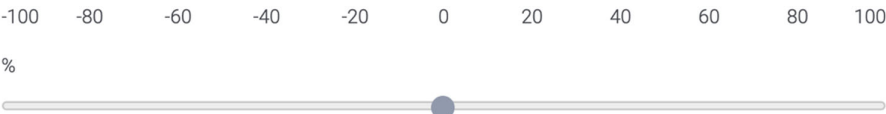
Suppose that the S&P500 index has increased by 15% over the past three months. How would you change your stock holdings.

e.g. if you would allocate 10% more of your wealth to the stock market, select 10%. If you would sell 10% of your stock market wealth, select -10%.



Suppose that the S&P500 index has increased by 20% over the past three months. How would you change your stock holdings.

e.g. if you would allocate 10% more of your wealth to the stock market, select 10%. If you would sell 10% of your stock market wealth, select -10%.



Please assign probabilities (from 0-100) to the following ranges of possible overall stock price changes (%) for the **S&P500 index** over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0	%
From 15% to 20%	0	%
From 10% to 15%	0	%
From 5% to 10%	0	%
From 0% to 5%	0	%
From -5% to 0%	0	%
From -10% to -5%	0	%
From -15% to -10%	0	%
From -20% to -15%	0	%
Less than -20%	0	%
Total	0	%

Please assign probabilities (from 0-100) to the following ranges of possible overall changes (%) for **your stock market portfolio** over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0	%
From 15% to 20%	0	%
From 10% to 15%	0	%
From 5% to 10%	0	%
From 0% to 5%	0	%
From -5% to 0%	0	%
From -10% to -5%	0	%
From -15% to -10%	0	%
From -20% to -15%	0	%
Less than -20%	0	%
Total	0	%

We would like to know what your opinion is about what **other investors** think will happen to the stock market price. Please assign probabilities (from 0-100) to the following ranges of beliefs that **other investors** might hold about overall price changes in the S&P 500 index over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0 %
From 15% to 20%	0 %
From 10% to 15%	0 %
From 5% to 10%	0 %
From 0% to 5%	0 %
From -5% to 0%	0 %
From -10% to -5%	0 %
From -15% to -10%	0 %
From -20% to -15%	0 %
Less than -20%	0 %
Total	0 %

By what percentage do you think the earnings of the companies listed on S&P500 have changed overall over the past 12 months?

-30-25-20-15-10-5051015202530

%

- Shown to treatment group 1

We would now like to show you some information on the S&P 500 index.

Over the past 12 months, the earnings of the companies represented in the S&P500 index have increased by approximately 2%. This is lower than the average of around 7.5% annually over the past 10 years.

Please proceed to the next page.

- Shown to treatment group 2

We would now like to show you some information on the S&P 500 index.

Other investors participated in this survey on average believe that the 12-month return of S&P500 index from October 2023 to September 2024 would be 3.21%. This is lower than the average of an around 9% annual return on S&P500 over the past 10 years.

Please proceed to the next page.

Now we'd like you to think about what you perceive as the most pessimistic and most optimistic outlooks for the **S&P500 return** over the 12 months from October 2023 to September 2024. What do you think the lowest 12-month return might be for this time period and what do you think the highest might be? (please provide an answer as % per year).



Now we want to ask you to think about the chance of the **S&P500 return** you entered in the previous question. Please assign a percentage chance to each return to indicate how likely you think it is that this return will actually happen to S&P500 index over the 12 months from October 2023 to September 2024.

Note: your answers have to be greater than or equal to 1%, where 1% means nearly no chance that this growth rate will happen. The sum should total to 100%.

S&P500 return will be -50%	0 %
S&P500 return will be 0%	0 %
S&P500 return will be 50%	0 %
Total	0 %

Now we'd like you to think about what you perceive as the most pessimistic and most optimistic outlooks for your stock market portfolio over the 12 months from October 2023 to September 2024. What do you think the lowest 12-month return might be for this time period and what do you think the highest might be? (please provide an answer as % per year).

-100 -80 -60 -40 -20 0 20 40 60 80 100

Lowest return (%)



Most likely return (%)



Highest growth (%)



Now we want to ask you to think about the chance of the the returns of **your stock market portfolio** you entered in the previous question. Please assign a percentage chance to each return to indicate how likely you think it is that this return will actually happen to S&P500 index over the 12 months from October 2023 to September 2024.

Note: your answers have to be greater than or equal to 1%, where 1% means nearly no chance that this growth rate will happen. The sum should total to 100%.

S&P500 return will be -70%	0 %
S&P500 return will be -46%	0 %
S&P500 return will be -25%	0 %
Total	0 %

Now we'd like you to think about what your opinion is about what **other investors** perceive as the most pessimistic and most optimistic outlooks for the **S&P500 return** over the 12 months from October 2023 to September 2024. What do you think that **other investors** would say the lowest 12-month return might be for this time period and what do you think the highest might be? (please provide an answer as % per year).

-100 -80 -60 -40 -20 0 20 40 60 80 100

Lowest return (%)



Most likely return (%)



Highest return (%)



Now we want to ask you to think about the chance of the the returns of **S&P500** index as **perceived by other investors** you entered in the previous question. Please assign a percentage chance to each return to indicate how likely you think it is that this return will actually happen to S&P500 index over the 12 months from October 2023 to September 2024.

Note: your answers have to be greater than or equal to 1%, where 1% means nearly no chance that this growth rate will happen. The sum should total to 100%.

S&P500 return will be -29% %

S&P500 return will be -13% %

S&P500 return will be 11% %

Total %

The next question is about the following problem. In questionnaires like ours, sometimes some participants do not carefully read the questions and just quickly click through the survey. This means that there are a lot of random answers which compromise the results of research studies. To show that you read our questions carefully, please select other as your answer to the next question. What is your favorite color?

green

blue

yellow

red

black

white

other

Each of the following 4 texts is sent to a random 25% of participants

For the next three questions, suppose that you get news that the S&P500 index would increase by 5% over the 12 months between October 2023 to September 2024.

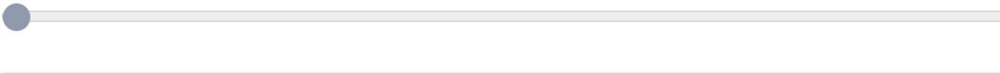
For the next three questions, suppose that you get news that the S&P500 index would increase by 10% over the 12 months between October 2023 to September 2024.

For the next three questions, suppose that you get news that the S&P500 index would increase by 15% over the 12 months between October 2023 to September 2024.

For the next three questions, suppose that you get news that the S&P500 index would increase by 20% over the 12 months between October 2023 to September 2024.

by what percentage would you change your wealth allocated to the stock market?

-100 -80 -60 -40 -20 0 20 40 60 80 100
%



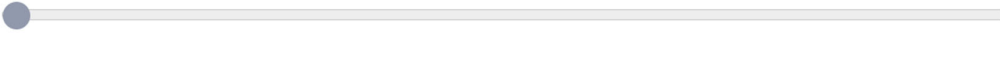
by what percentage do you think **other investors** would change their wealth allocated to the stock market?

-100 -80 -60 -40 -20 0 20 40 60 80 100
%



by what percentage would you change your wealth allocated to the stock market if **other investors** do not change how much they would allocate to the stock market?

-100 -80 -60 -40 -20 0 20 40 60 80 100
%



This question is being asked to all participants in the survey, drawn from a representative sample of stock investors in the US. Please choose a number from 1 to 100. We will take your number as well as the numbers chosen by **other investors** to calculate the average pick. The winning number will be the number that is closest to two-thirds ($2/3$) of the average. If your number is the winning number, you will receive a bonus payment of 20 dollars.

1 11 21 31 41 51 60 70 80 90 100

number

Other investors are also asked to guess a number from 1 to 100, with the goal of making their guess as close as possible to two-thirds of the average guess of all those participating in the contest. What percentage (%) of **other investors'** guesses do you think will fall in each of the following ranges?

1-10	0 %
11-20	0 %
21-30	0 %
31-40	0 %
41-50	0 %
51-60	0 %
61-70	0 %
71-80	0 %
81-90	0 %
91-100	0 %
Total	0 %

b. Second Wave Survey

What is the total level (\$) of your current wealth?

Note: wealth includes checking/saving accounts, pensions/retirement, brokerage account, real estate assets, and other assets.

0 - 2,500

2,500 - 5,000

5,000 - 7,500

7,500 - 10,000

10,000 - 25,000

25,000 - 50,000

50,000 - 75,000

75,000 - 100,000

100,000 - 250,000

250,000 - 500,000

500,000 - 1,000,000

1,000,000 - 2,500,000

2,500,000 - 5,000,000

> 5,000,000

Approximately what percentage of your current wealth is financial wealth?

Note: financial wealth includes stocks, ETFs, financial derivatives, bonds, pension funds, bank savings, and other wealth in the financial system.

0 10 20 30 40 50 60 70 80 90 100
%



We would now like to ask how your current financial assets (excluding real estate) are distributed across different asset classes. Please enter the approximate percentage you currently have invested in the following asset classes.

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

Stocks (Individual Companies)	0	%
ETFs or index fund	0	%
Financial derivatives (option, future, forward, etc)	0	%
Bonds	0	%
Pension fund (401k, IRA etc)	0	%
Other	0	%
Total	0	%

By how much has the value of your stock market portfolio changed in percentage over the **past three months**?

Note: please use a negative value for a decrease and a positive value for an increase.

-100 -80 -60 -40 -20 0 20 40 60 80 100
%



Please assign probabilities (from 0-100) to the following ranges of possible overall stock price changes (%) for the **S&P500 index** over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0	%
From 15% to 20%	0	%
From 10% to 15%	0	%
From 5% to 10%	0	%
From 0% to 5%	0	%
From -5% to 0%	0	%
From -10% to -5%	0	%
From -15% to -10%	0	%
From -20% to -15%	0	%
Less than -20%	0	%
Total	0	%

Please assign probabilities (from 0-100) to the following ranges of possible overall changes (%) for **your stock market portfolio** over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0	%
From 15% to 20%	0	%
From 10% to 15%	0	%
From 5% to 10%	0	%
From 0% to 5%	0	%
From -5% to 0%	0	%
From -10% to -5%	0	%
From -15% to -10%	0	%
From -20% to -15%	0	%
Less than -20%	0	%
Total	0	%

We would like to know what your opinion is about what **other investors** think will happen to the stock market price. Please assign probabilities (from 0-100) to the following ranges of beliefs that **other investors** might hold about overall price changes in the S&P 500 index over the 12 months from October 2023 to September 2024:

Note: the sum of the answers has to be equal to 100%. Answers can range from 0% to 100%.

More than 20%	0 %
From 15% to 20%	0 %
From 10% to 15%	0 %
From 5% to 10%	0 %
From 0% to 5%	0 %
From -5% to 0%	0 %
From -10% to -5%	0 %
From -15% to -10%	0 %
From -20% to -15%	0 %
Less than -20%	0 %
Total	0 %

The next question is about the following problem. In questionnaires like ours, sometimes some participants do not carefully read the questions and just quickly click through the survey. This means that there are a lot of random answers which compromise the results of research studies. To show that you read our questions carefully, please select other as your answer to the next question. What is your favorite color?

green
blue
yellow
red
black
white
other

Based on your experience and observations as a stock market investor, how many days do you believe it typically takes for **you** to react to significant news events in the stock market? Consider news events such as earnings reports, geopolitical developments, and macroeconomic data releases, etc.



Based on your experience and observations as a stock market investor, how many days do you believe it typically takes for **other investors** to react to significant news events in the stock market? Consider news events such as earnings reports, geopolitical developments, and macroeconomic data releases, etc.



What proportion of your pension fund is currently allocated to equity investments?

Note: If you don't have any pension fund, please select zero.

